1 Given that $\frac{3^x}{9^{3x}} = 81$

find the value of x.
Show clear algebraic working.

$9^{3x} = (3^2)^{3x}$

$= 3^{6x}$

$81 = 3^4$

$\therefore \frac{3^x}{9^{3x}} = 81 \rightarrow \frac{3^x}{3^{6x}} = 3^4$ (1)

$3^{x-6x} = 3^4$

$x - 6x = 4$ (1)

$-5x = 4$

$x = -\frac{4}{5} = -0.8$ (1)

$x =$ −0.8

(Total for Question 1 is 3 marks)

$$2^{2y} \times 2^{3y+2} = \frac{8^{5y}}{4^n}$$

2 (b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

$8^{5y} = (2^3)^{5y}$

$= 2^{15y}$ (1)

$4^n = (2^2)^n$

$= 2^{2n}$

$2^{2y} \times 2^{3y+2} = \frac{2^{15y}}{2^{2n}}$

$2^{2y+3y+2} = 2^{15y-2n}$ (1)

$2y + 3y + 2 = 15y - 2n$

$5y + 2 = 15y - 2n$ (1)

$2n = 15y - 5y - 2$

$2n = 10y - 2$

$n = 5y - 1$ (1)

$n = 5y - 1$

(4)

(Total for Question 2 is 4 marks)

3 $\sqrt{2} \times 16 = 2^x$

$\sqrt{2} = 2^{1/2}$

$16 = 2^4$

(a) Find the value of x.
Show your working clearly.

$2^{1/2} \times 2^4 = 2^x$ (1)

$2^{1/2+4} = 2^x$

$x = 4 + 1/2 = 4.5$ (1)

$x =$ 4.5

(2)

$$\frac{(11^{-6})^5}{11^4} = 11^n$$

(b) Find the value of n.
Show your working clearly.

$\frac{11^{-6\times5}}{11^4} = 11^n$

$\frac{11^{-30}}{11^4} = 11^n$ (1)

$11^{-30-4} = 11^n$

$n = -30 - 4 = -34$ (1)

$n =$ -34

(2)

(Total for Question 3 is 4 marks)

4 Simplify fully $\left(\frac{9x^4}{16y^{10}}\right)^{-\frac{1}{2}}$

$9x^4 = 3^2x^4$

$16y^{10} = 2^4y^{10}$

$\left(\frac{9x^4}{16y^{10}}\right)^{-\frac{1}{2}} = \left(\frac{3^2x^4}{2^4y^{16}}\right)^{-\frac{1}{2}}$

$= \frac{3^{2(-\frac{1}{2})} \cdot x^{4(-\frac{1}{2})}}{2^{4(-\frac{1}{2})} \cdot y^{10(-\frac{1}{2})}}$ (1)

$= \frac{3^{-1} \cdot x^{-2}}{2^{-2} \cdot y^{-5}}$ (1) $= \frac{2^2 \cdot y^5}{3x^2}$

$= \frac{4y^5}{3x^2}$ (1)

$\frac{4y^5}{3x^2}$

(Total for Question 4 is 3 marks)

5 (a) Simplify $8^2 \times \sqrt[3]{4^6}$

Give your answer in the form 2^a where a is an integer.

Show each stage of your working clearly.

$= 8^2 \times \sqrt[3]{4^6}$

$= (2^3)^2 \times (4^6)^{\frac{1}{3}}$

$= 2^6 \times 4^2$ (1)

$= 2^6 \times (2^2)^2$

$= 2^6 \times 2^4$ (1)

$= 2^{(6+4)}$

$= 2^{10}$ (1)

2^{10}

(3)

Given that $n^{\left(-\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^4$ where $n > 0$

(b) find the value of n.

$n^{\left(-\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^4$

$\frac{1}{n^{\frac{4}{5}}} = \frac{1}{16}$ (1)

$16 = n^{\frac{4}{5}}$

$16^{\frac{5}{4}} = n$ (2)

$n = 32$ (1)

$n =$ 32

(4)

(Total for Question 5 is 7 marks)

6 $\dfrac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$\dfrac{2^k}{2^{2n}} = 2^x$ (1)

$2^{k-2n} = 2^x$

$x = k - 2n$ (1)

$x =$ $k - 2n$

(Total for Question 6 is 2 marks)

7 Given that $\left(\sqrt[3]{\frac{1}{x}}\right)^4 = x^m$

(a) find the value of m

$(x^{-1})^{4/3} = x^{-\frac{4}{3}}$

$m = -\frac{4}{3}$

$m =$ $-\frac{4}{3}$ (1)

(1)

(Total for Question 7 is 1 marks)

8 $a = 6 \times 10^{40}$

Work out the value of a^3
Give your answer in standard form.

$a^3 = 6^3 \times (10^{40})^3$

$= 216 \times 10^{120}$ (1)

(1)

$= 2.16 \times 10^{122}$ (1)

2.16×10^{122}

(Total for Question 8 is 3 marks)

9 Solve $2^{-4x} = 32$

$2^{-4x} = 2^5$

$-4x = 5$ (1)

$x = -\frac{5}{4}$ (1)

$x =$ $-\frac{5}{4}$

(Total for Question 9 is 2 marks)

10 Given that

$$2^n = 2^{x^2} \times 16^x \times 8$$

and

$$x > 0$$

find an expression for x in terms of n
State any restrictions on n

$16 = 2^4$ (1)

$8 = 2^3$

$2^n = 2^{x^2} \times 2^{4x} \times 2^3$

$n = x^2 + 4x + 3$ (1)

$n = (x+2)^2 - 4 + 3$

$n = (x+2)^2 - 1$ (1)

$n+1 = (x+2)^2$

$\pm\sqrt{n+1} = x + 2$

$x = -2 \pm \sqrt{n+1}$ (1)

since $x > 0$, $-2 + \sqrt{n+1}$ only

let $x = 0$, $0 = -2 + \sqrt{n+1}$

$2 = \sqrt{n+1}$

$4 = n+1$

$n = 3$

$\therefore n > 3$ for $x > 0$

$x = -2 + \sqrt{n+1}$ and $n > 3$. (1)

(Total for Question 10 is 5 marks)

11 Express $\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}}$ in the form $\frac{m^ak^b}{c}$ where a, b and c are integers to be found.

$\frac{m^{6(\frac{3}{2})} \times k^{10(\frac{3}{2})}}{25^{3/2}} = \frac{m^9 \times k^{15}}{125}$

$= \frac{m^9 k^{15}}{125}$

(2)

$\frac{m^9 k^{15}}{125}$

.....

(Total for Question 11 is 2 marks)

12 (a) $\sqrt{2} \div \dfrac{8^3}{16^{\frac{3}{2}}} = 2^n$

Work out the value of n
Show your working clearly.

$\sqrt{2} = 2^{\frac{1}{2}}$

$8^3 = (2^3)^3 = 2^9$

$16^{\frac{3}{2}} = (2^4)^{\frac{3}{2}} = 2^6$ (1)

$2^{\frac{1}{2}} \div \dfrac{2^9}{2^6} = 2^n$

$2^{\frac{1}{2}-(9-6)} = 2^n$ (1)

$\frac{1}{2} - 3 = n$ $\quad n = -2.5$ (1)

$n =$ -2.5

(3)

(b) Find 4% of 4.5×10^{157}
Give your answer in standard form.

$0.04 \times 4.5 \times 10^{157}$ (1)

$= 4 \times 10^{-2} \times 4.5 \times 10^{157}$

$= 4 \times 4.5 \times 10^{-2+157}$

$= 18 \times 10^{155}$ (1)

$= 1.8 \times 10^{156}$ (1)

1.8×10^{156}

(3)

(Total for Question 12 is 6 marks)

13 (a) Simplify fully $\left(32a^{15}\right)^{\frac{3}{5}}$

$32^{\frac{3}{5}} \times a^{15\left(\frac{3}{5}\right)}$

$= 8 \times a^{9}$

$8a^{9}$ (2)

.....

(2)

(b) Express $\left(\frac{1}{10x}\right)^{-3}$ in the form px^n where p and n are integers.

$((10x)^{-1})^{-3} = (10x)^{3}$

$= 1000x^{3}$

$1000x^{3}$ (2)

.....

(2)

(c) Solve $\frac{1-2y}{3} = \frac{4}{5} - \frac{2y-1}{2}$

Show clear algebraic working.

$(5)(2)(1-2y) = 4(3)(2) - (3)(5)(2y-1)$ (1)

$10 - 20y = 24 - 30y + 15$ (1)

$10y = 29$

$y = 2.9$ (1)

$y =$ 2.9

.....

(3)

(Total for Question 13 is 7 marks)